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A Scholar in His Time: Contemporary views of Kosambi the mathematician

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A Scholar in His Time: Contemporary views of Kosambi the mathematician*

Ramakrishna Ramaswamy

"Kosambi introduced a new method into historical scholarship, essentially by application of modern mathematics." J.D. Bernal¹, who shared some of his interests and much of his politics, summarized the unique talents of D.D. Kosambi² in an obituary that appeared in the journal Nature, adding, "Indians were not themselves historians: they left few documents and never gave dates. One thing the Indians of all periods did leave behind, however, were hoards of coins. [...] By statistical study of the weights of the coins, Kosambi was able to establish the amount of time that had elapsed while they were in circulation ... "

In brief, the facts of DDK's academic life are as follows. He attended high-school in the US, in Cambridge, MA, and undergraduate college at Harvard, graduating in 1929. On returning to India, he worked as a mathematician at Banaras Hindu University (1930–31), Aligarh Muslim University (AMU) (1931–33), Fergusson College, Pune (1933–45), and the Tata Institute of Fundamental Research (TIFR) (1945–62), after which he held an emeritus fellowship of the Council of Scientific and Industrial Research (CSIR) until his death at the age of 59, in 1966.

Today the significance of D.D. Kosambi's mathematical contributions^{3–71} tends to be underplayed, given the impact of his scholarship as a historian and Indologist. Although his work in the latter areas has been collected in several volumes⁷² and critical commentaries have appeared over the years^{73, 74}, his work in mathematics has not been compiled and reviewed to the same

^{*} Revised version of the Public Lecture delivered at the Nehru Memorial Museum and Library, New Delhi on 31 July 2014, D.D Kosambi's 106th birth anniversary.

extent^{75, 76, 77, 78}. Indeed, a complete bibliography is not available in the public domain so far⁷⁹. This asymmetry is unfortunate since, as commented elsewhere⁷⁵, an understanding of Kosambi the historian can only be enhanced by an appreciation of Kosambi the mathematician⁸⁰.

DDK is known for several contributions, some of which like the Kosambi-Cartan-Chern (KCC) theory⁸¹, carry his name, and some like the Karhunen–Loève expansion^{37, 39, 82} that do not. The Kosambi mapping function in genetics⁴⁰ continues to be used to this day⁸³, but the path-geometry that he studied for much of his life⁸⁴ has not found further application. DDK's final years were mired in controversies, both personal and professional. His papers on the Riemann hypothesis (RH)^{65, 66} brought him a great deal of criticism, while his personal politics put him in direct conflict with Homi Bhabha and the Department of Atomic Energy. This contributed to his eventual and somewhat ignominious ouster from employment at the TIFR. His early and passionate advocacy of solar energy was practical and based on sound scientific common sense. In some of his arguments, he seems even somewhat Gandhian, and although this was a contrary position to hold in the TIFR at that time, the essential validity of his argument remains to this day⁸⁵.

DDK was just about 23 years old when he returned to India and took up a temporary position at the Banaras Hindu University with a BA (summa cum laude) from Harvard. A year later he moved to the Aligarh Muslim University where he was appointed in the Mathematics Department at the suggestion of André Weil⁸⁶ who, just about a year older, was then already well known as a mathematician and as a prodigy, and who had been invited to the AMU by the then Vice Chancellor Syed Ross Masood.

Although Weil did not last long in Aligarh, his influence on Kosambi was considerable. In addition to giving him the position and encouraging him on the matter of the Bourbaki prank,⁸⁶ Weil helped DDK forge early mathematical links with, among others, T. Vijayaraghavan⁸⁷ and S. Chowla⁸⁸. He undoubtedly influenced his taste in mathematics, possibly sparking DDK's interest in the Riemann hypothesis. Weil

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would, in the early 1940s make important contributions to this field⁸⁹ although when DDK turned to it almost 30 years later⁶⁵ his efforts were to come a cropper. Weil spent the summer of 1931 in Europe and upon his return to Aligarh, he found that not only had his own position been compromised but the group of mathematicians that he had put together had also fragmented, with Vijayaraghavan having moved to take up a Professorship in Dacca.⁹⁰ By early 1932, Weil had returned to Europe, and DDK was to leave Aligarh soon thereafter.

Kosambi started his independent work in Aligarh, choosing the area of *path-geometry*, a term he coined, submitting his papers to leading European journals^{7, 9, 10}. One that was sent to *Mathematische Zeitschrift* was also communicated to Elie Cartan who was inspired enough by the result to write a detailed commentary which included an extract of the correspondence that Kosambi had with him. This was also published in *Mathematische Zeitschrift*¹¹ immediately following DDK's paper in 1933. Along with a later paper by the Chinese mathematician, S.S. Chern, these three works constitute what is now termed the KCC-theory, a topic that has, in recent years, found new applications in physics and biology⁸¹. Some years later, in 1946, Kosambi tried to have Chern invited to visit India when he was at the TIFR but nothing came of it.

DDK wrote many papers on path-geometry, and in the mid-1940s summarized his work in a manuscript that was submitted to Marston Morse at the Institute for Advanced Study in Princeton. In a letter⁹¹ to Bhabha he says, "The book on Path-Geometry will, according to a letter from Morse, appear in the *Annals of Mathematics Studies*, Princeton." This book was never published—indeed very few books in this series were, and efforts to locate a copy of the manuscript in the Morse archives have proved fruitless.⁹² DDK makes reference to a second copy of the manuscript that he gave to Bhabha, but that copy has not been located either.

The Nobel laureate, C.V. Raman visited Aligarh in 1931 as the member of a selection committee, and although there is no specific record of his having met Kosambi, his subsequent actions suggest that he quickly formed, either directly or indirectly, a very high opinion of

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DDK. In 1934, when Raman founded the Indian Academy of Sciences (IASc) in Bangalore, he elected Vijayaraghavan and Chowla. The very next year Kosambi was elected to the IASc at the age of 28, when his mathematical œuvre was slight, and along with others such as P.C. Mahalanobis and V.V. Narlikar. Kosambi was one of the younger of the Founding or Foundation Fellows (namely those elected in 1934 and 1935). Since the initial election to the Academy was almost entirely his decision, the estimation that Raman had of Kosambi's scholarship or of his potential, must have been considerable. It is possible that Vijayaraghavan may have played some role in this early recognition⁹³, and it is also likely that the award of the first Ramanujan Prize of the Madras University in 1934 to S. Chandrashekar, S. Chowla and DDK⁹⁴ would have favourably impressed Raman. As it happened, in later years Kosambi was privately and publicly very critical of Raman's style of functioning⁹⁵.

This early recognition, however, stood him in good stead. He published a couple of papers in the academy's journal, *Proceedings of the Indian Academy of Sciences* in 1935 (and not again until the 1960s when, as S. Ducray, he published two more). Reviews of his papers in other journals began to appear in *Current Science*, the general science journal started by Raman, in addition to original articles that he chose to publish in this journal as well. Indeed all his initial papers on the quantitative approach to numismatics^{26, 27, 34, 36} appeared in *Current Science*.

1. Reviews and Commentaries

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One of the early references to the work of DDK on numismatics that was brought to the attention of readers of *Current Science* was a review in 1941⁹⁶ by K.A.N. (this was probably the well-known historian K.A. Nilakantha Sastry) of two papers of DDK in the *New Indian Antiquary*⁹⁷. By this time, DDK seems to have been well established as an eminent mathematician. While generally admirable of the work, K.A.N. comments on a number of DDK's characteristics: the use of "hard phrases" in his critique of the methods used by others, his exposure "of the hollowness of much pseudo-expertise that has held the field", etc. Nevertheless, the review is not uniformly accepting

of DDK's conclusions, and K.A.N. does alert the reader to potential areas of inaccuracy. In a charming final paragraph, for instance, he says,

Yet, this conclusion hardly tallies with the impressions of the Mauryan epoch gathered from other sources like the inscriptions of Asoka, or the polished stone pillars—not to speak of Megasthenes and the *Arthasastra*. There are other statements, *obiter dicta*, which may surprise the reader, and even shock him; but there is much, very much in these papers and their method for which he will be grateful.

The journal *Mathematical Reviews (MR)* was started in 1940 by the American Mathematical Society as a way for working mathematicians to keep up with the increasing numbers of papers that appeared each year in diverse journals. The practice was (and still is) to provide a brief summary of these papers—sometimes with commentary, and sometimes without. Indeed, some papers are merely noted or abstracted, and almost all reviews are signed.

Of DDK's 60 or so papers in mathematics, about half were reviewed in *MR*; these are indicated in the References^{3–71}. The reviewers include R.L. Anderson, R.P. Boas, Jr., N. Coburn, J.L. Doob, W. Feller, V. Hlavaty, M. Janet, A. Kawaguchi, J.B. Kelly, M.S. Knebelman, J. Korevaar, J. Kubilius, R.G. Laha, W.J. LeVeque, A. Nijenhuis, E.S. Pondiczery (a pseudonym of R.P. Boas Jr.), A. Rényi, J.A. Schouten, E.W. Titt, J.L. Vanderslice, O. Varga, B. Volkmann, A. Wald, and J. Wolfowitz. Several of these reviews are just summaries of the papers, but some are serious commentaries on the work of Kosambi, and, significantly, are by some of the leading contemporaneous mathematicians, probabilists, and statisticians. Indeed R.P. Boas Jr. who reviewed some of the papers was one of the main editors of *MR*.

It may be pertinent to note that it is not just DDK's papers that were published in journals outside India that were reviewed in *MR*; several of the papers published in Indian journals were also commented upon critically. These include the important paper, "Statistics in function

space"³⁹ that was reviewed by the probabalist, J.L. Doob, who went on to become the President of the American Mathematical Society (and who was awarded the National Medal of Science by the President of the United States, Jimmy Carter in 1979).

Although Doob gave a careful and comprehensive review of the work soon after it was published in 1943, unfortunately neither Karhunen nor Loève who essentially rediscovered these results⁸² were aware either of the paper or of its review, and today these results go under their names, and Kosambi's contribution is largely unrecognized. One important feature of the paper pointed out in the review,

The author discusses statistical problems connected with continuous stochastic processes whose representative functions x(t) [. . .] Various mechanical and electrical methods are suggested for combining functions x(t), given graphically, as necessitated by this type of statistical approach.

was the idea of a mechanical or electrical computer. This was to be part of Kosambi's *Kosmagraph* project that was in part funded by a grant from the J.R.D. Tata Trust in 1945. It is not clear if a working model was ever successfully constructed, though there is a reference to it in a report he sent to the Tata Trust⁸⁰,

The *Kosmagraph* is finished, and a working model being improved at St. Xavier's College. The total outlay for workshop charges, electric motors, cathode ray oscillographs, valve tubes etc. would have exceeded the total amount of the Tata grant. But the St. Xavier's authorities stood the expense of these items, as Fr. Rafael has collaborated in the work. My total expenses from the grant have been a nominal honorarium of Rs.250/- to K.B. McCabe, the third collaborator; and another of Rs.50 to Salvador D'Souza, head mechanic at the St. Xavier's workshop. Both have deserved far more, and the work of McCabe in particular seems to me to be beyond recompense.

A joint paper is being made ready for publication, though it will be some months before all the points are checked.

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The paper alluded to does not appear to have been published, and no drafts have been located among DDK's papers. It is also not clear what became of the project; the interest in a computing machine stayed with Kosambi when he later moved to TIFR, and indeed was one of reasons of his visit to the USA in 1948–49⁷⁵.

Another of DDK's reviewers was Abraham Wald (who was later to die in a plane crash in India when he was visiting the country at the invitation of the Indian government) who commented, generally favourably, on four of his papers. What is interesting is that many of the papers were published in journals such as *Mathematics Student* and the *Journal of the Indian Mathematical Society*, both of limited circulation, and which to this day remain somewhat difficult to locate⁹⁸.

It should be mentioned that most of DDK's publications in mathematics are independently authored. He did, however, mentor several students, both formally and informally at the TIFR in the 1950s, and among these were S. Raghavachari and U.V. Ramamohana Rao who are his only co-authors.

2. The RH papers

Arguably the most important as yet unresolved problem in pure mathematics is a hypothesis that was enunciated in 1857 by the celebrated mathematician, Bernhard Riemann. A brief introduction to the nature of the mathematical problem⁹⁹ is included here for those who are less acquainted with it, to give some flavour of why it is interesting and a challenge. (The author would like to mention here that at the risk of losing half the potential readership with each equation¹⁰⁰, it is absolutely essential that some be retained. For all of mathematics there is no greater game than to solve the RH, and to appreciate both what Kosambi tried, and where he did not succeed, some equations are needed).

The RH concerns properties of a mathematical function that has been studied for at least four centuries. This is the zeta function, the sum of inverse powers of the integers,

$$\zeta(z) = 1 + \frac{1}{2^z} + \frac{1}{3^z} + \frac{1}{4^z} + \dots,$$

the ellipsis signifying that the sequence does not terminate. When *z* is equal to zero, then each term is 1, and the sum, namely $1+1+1+1 \dots$ becomes infinitely large: $\zeta(0) \rightarrow \infty$.

Such infinite sums have long been of interest: an example that will be familiar to many is the sum that arises in Zeno's paradox regarding Achilles and the tortoise. (In a 100-metre race, the tortoise, which is 100 times slower than Achilles is given, say, a head-start of 90 m. In the time that Achilles covers 90 m, the tortoise covers 90 cm and is therefore still ahead. In the time that Achilles covers the 90 cm, the tortoise goes ahead by 9 mm, when Achilles covers the 9 mm, the tortoise is ahead by a smaller fraction, and so on. So Achilles would, it seems, never catch up with the tortoise. The resolution of the paradox is that this infinite sum is actually a finite quantity, and Achilles wins the race easily¹⁰¹).

The harmonic series, $\zeta(1)$ is the sum of inverses of the integers,

$$\zeta(1) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots \rightarrow \infty$$

and this also diverges or becomes infinite (the arrow symbol \rightarrow in the equation below signifies "tends to"). In contrast, when z=2, the sum is a finite number,

$$\zeta(2) = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots \to \frac{\pi^2}{6}.$$

Clearly the value of the zeta function,

$$\zeta(z) = \sum_{n=1}^{\infty} \frac{1}{n^{z}}$$

depends on the value of z, and Riemann was interested in its "zeroes", namely those values of z when $\zeta(z) = 0$

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In order to study the general properties of such a function, it is necessary to consider all possible values for *z*, in particular when *z* is a *complex* number, namely of the form z = x + iy where *x* is the real part and *y* the imaginary part, and $i = \sqrt{-1}$. It turns out then that the ζ function can take values that are either positive or negative depending on the value of *z*, or equivalently, on the values of *x* and *y*. When *y* = 0 and *x* is a negative even integer, namely -2, -4, -6 and so on, the function takes the value 0: these are termed "trivial" zeroes since the function can be shown to vanish through a straightforward procedure¹⁰².

The ζ function has in addition an infinite number of "nontrivial" zeros, and RH is that for *all* of these, *x* (namely the real part of *z*) has the value 1/2. In the complex plane, these zeroes therefore all lie on the so-called "critical" line, *x* = 1/2. While being simple enough to state, it remains unproven to this day. Because of connections between the zeta function and prime numbers, a proof of the RH would have significant implications for the distribution of prime numbers, and via this, for much of mathematics. An alternate measure of its importance can be gauged from the fact that it is one of the so-called Millennium Prize problems for which the Clay Mathematical Institute has announced a grand cash award in the past decade.

DDK's mathematical reputation suffered greatly as a result of two papers he published in the *Journal of the Indian Society of Agricultural Statistics (JISAS)*^{63, 67}; in one of them, he claimed a result that was essentially a proof of the RH. Notwithstanding its name, the journal does publish serious mathematics, particularly in the area of probability. Although obscure and highly specialized, the journal may not have been as inappropriate for the papers as might appear since the methods suggested by Kosambi were probabilistic. However, it is not clear that the journal had a proper peer–review process in place whereby submitted articles would, prior to publication, be examined by experts in the field. The lack of appropriate reviewing was a real deficiency, more so for a claim of this magnitude and the charge remains that DDK chose to publish the papers in *JISAS* to be able to pass off a doubtful "proof".

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Both papers were reviewed subsequently in MR, one by W.J. LeVeque, a number theorist who eventually became Executive Director of the American Mathematical Society. His critique of "An application of stochastic convergence"63 goes straight to the point that the claim made by DDK "is a result which easily implies the Riemann hypothesis". However, since the proof is probabilistic in nature, there are major problems that he identifies: "Of the two proofs given for the crucial Lemma 1.2, the reviewer does not understand the first, which seems to involve more 'hand-waving' than is customarily accepted even in proofs of theorems less significant than the present one. The second proof appears to be erroneous." The review concludes "The reviewer is unable either to accept this proof or to refute it conclusively. The author must replace verbal descriptions, qualitative comparisons and intuition by precise definitions, equations and inequalities, and rigorous reasoning, if he is to claim to have proved a theorem of the magnitude of the Riemann hypothesis."

The kindest analysis of these works of DDK comes from the Hungarian mathematician, A. Rényi who says in a posthumous review of the paper "Statistical methods in number theory"⁶⁷ that

The late author tried in the last 10 years of his life to prove the Riemann hypothesis by probabilistic methods. Though he did not succeed in this, he has formulated the following highly interesting conjecture on prime numbers.

Rényi, who had been sent both these and the earlier papers^{63, 64} prior to publication, goes on to say that "Neither in this paper nor in his previous paper [*Proc. Nat. Acad. Sci. U.S.A.* **49** (*1963*), *20–23;* MR0146168 (26 #3690)] did the author succeed in proving his hypothesis, nor in deducing from it the Riemann hypothesis." The *PNAS* paper⁶⁴ was reviewed by J.B. Kelley who states, after summarizing the main result, that "The exposition is rather sketchy; in particular, the reviewer could not follow the proof of the crucial Lemma 4."

Either because of the timing of the review or because he may have appreciated the valiant attempts of DDK to prove the RH by an unusual route, Rényi, concludes the review by saying that at that point in time namely in (1968),

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one does not have enough knowledge of the fine structure of the distribution of primes to prove or disprove the author's conjecture. The problem seems to be even more difficult than the problem of the validity of the Riemann hypothesis. As a matter of fact, no obvious method exists to prove the author's hypothesis even under the assumption of the Riemann hypothesis. Nevertheless, the conjecture is worthy of study in its own right, and the reviewer proposes to call it "the Kosambi hypothesis" in commemoration of the enthusiastic efforts of the late author.

Rényi's suggestion has not found favour. The probabilistic approach has inherent limitations, as the physicist Michael Berry points out¹⁰³. Indeed, as these reviews suggest, the rigour emphasised by DDK in his early years had deserted him. What is somewhat surprising is that there are elementary errors in these papers that become evident even with a fairly cursory examination, and which could have been detected by an alert referee. The fact that *JISAS* published this paper with the errors added to the feeling that DDK deliberately chose the journal to avoid a qualified peer review. These papers essentially destroyed DDK's mathematical reputation.

Given the ongoing interest in the RH, only in part increased by its inclusion as a Millennium Prize problem, there are a number of popular books¹⁰⁴ that summarize the approaches to proving it. Not surprisingly, the work of DDK is not mentioned, although Berry remarks¹⁰³ that "his idea for proving RH based on showing that a certain function is nonsingular off the line, is ingenious". Andrew Odlyzko, another mathematician who has worked extensively on the RH says¹⁰⁵ that he "was really intrigued by these approaches, but after a while decided that it would take some clever insights far beyond what [he] could think of to accomplish anything rigorous in this area". Among Odlyzko's major contributions to a study of the RH is the computation of a large number of zeros (several million of them, in fact) to fairly high precision; for all of these, the real part equals 1/2. As an experimental mathematician he has a good insight into the approach suggested by DDK, adding, "In summary, I think it is a pity that Kosambi did not see the flaws in his arguments and published this paper, but the basic idea is an interesting one, and certainly worth exploring. I would be

surprised, but not shocked, if somebody clever managed to do something with it."

3. Bhabha and DDK

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DDK joined the newly formed TIFR on 16 June 1945. His appointment, which was for an initial period of five years was decided at the first meeting of the provisional council of TIFR.

The initial correspondence between Bhabha and DDK, although formal, was extremely cordial⁷⁴. In 1946, when Bhabha travelled to England, he appointed DDK Acting Director, leaving him in charge of the fledgling institute. This was a position of considerable responsibility, and one that DDK clearly enjoyed, and in a long letter⁹¹ dated 8th July he writes "About building up a School of Mathematics in India, we also think alike; but, as you are fully aware, we have to get people trained in a considerable number of branches for which there are no real specialists in this country."

The relationship also grew warm, especially since they had to plan the Institute together, concerning themselves with details regarding land acquisition, equipping the laboratories, hiring staff and planning for the future. That same year DDK was elected Fellow of the Indian National Science Academy, and the next, in 1947, was awarded the Bhabha Prize (named for Bhabha's father, Jehangir Hormusji Bhabha). He was also chosen President of the Mathematics section of the 34th Indian Science Congress that was held in Delhi in December 1947⁴⁷ with the active support of Bhabha who also realized that this would bring DDK into contact with Nehru. Kosambi's mathematical and statistical expertise was also greatly appreciated. A number of colleagues, Bhabha among them, acknowledged his advice and helped explicitly in their scientific publications.

In 1948, when DDK was to go to the US for a year's visit, to Chicago and Princeton, Bhabha threw a party for him at his residence in Malabar Hill. This visit was in fact largely arranged by Bhabha and, among other things, DDK was to investigate the possibility of getting a computing machine for the new institute⁹¹ and also attract new faculty,

K. Chandrasekharan and S. Minakshisundaram in particular. On this trip, he pursued all aspects of his wide-ranging interests, visiting Einstein and von Neumann in Princeton, Norbert Wiener in Boston, as well as the historian, A.L. Basham in London. In Chicago, he was a visiting Professor at the University, where he gave a course of 36 lectures on tensor analysis. This was a special interest to him: he had been invited to the editorial board of the Hokkaido University journal, *Tensor* (New Series), and indeed an article of his had already been translated into Japanese in 1939 by the same journal²⁵.

Chandrasekharan joined the TIFR in 1950 or so, and shortly thereafter, so did K.G. Ramanathan, who had obtained his PhD at Princeton. They were to play a much more influential role in shaping the TIFR School of Mathematics. In the next few years, though, the cracks in the relationship between Bhabha and DDK surfaced, first in regard to students and then gradually, with regard to details such as his attendance in office and other aspects of his working.

The spiral downwards, though, began in 1959 with the publication of the *JISAS* paper⁶³ and the subsequent grand obsession with a probabilistic proof of the RH. His differences became more pronounced with Bhabha who relied more and more on Chandrasekharan's opinion and estimation of DDK's work. The *coup de grace* was a letter signed by four of the mathematicians at TIFR stating that Kosambi had become an embarrassment to the Institute with his claim of the proof of the RH and of Fermat's Last Theorem¹⁰⁶ that was being broadcast internationally.

There were other differences with Bhabha which were of a political nature, but these differences were already present in 1946 when Bhabha invited DDK to join TIFR. The unpublished (and largely unknown) essay "An Introduction To Lectures On Dialectical Materialism" relates to a set of 15 lectures given by Kosambi to the citizenry of Pune in 1943. Later, when he gave a set of lectures on Statistics at TIFR the notes conclude with an appreciation of Lenin¹⁰⁷. Indeed, Bhabha facilitated DDK's visits to the Soviet Union and China, and it is not possible that DDK's views were hidden under a bushel until the early 1960s.

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In July 1960 DDK gave a talk to the Rotary Club of Poona on "Atomic Energy for India". This essay¹⁰⁸ is an unabashed advocacy of solar power over atomic power, mirroring in a sense his ideological conflict with the Department of Atomic Energy (DAE). Half a century later, many of these issues remain current and the arguments remain valid, as for example the following observation

It seems to me that research on the utilization of solar radiation, where the fuel costs nothing at all, would be of immense benefit to India, whether or not atomic energy is used. But by research is not meant the writing of a few papers, sending favoured delegates to international conferences and pocketing of considerable research grants by those who can persuade complaisant politicians to sanction crores of the taxpayers' money. Our research has to be translated into use.

There is more in these essays on solar energy that merits attention even today such as his observations on energy storage and distribution, and on environmental issues¹⁰⁸. Eventually matters came to such a pass as to cause the DAE to not renew DDK's contract. As already pointed out, the RH papers had caused a serious blow to Kosambi's mathematical reputation and while this was made out as the proximate cause for his dismissal from TIFR, trouble had been brewing for some time. The letters between Bhabha and DDK grew increasingly formal, bureaucratic, and strained. There was a distinct difference in styles, and the iconoclastic Kosambi was hardly one to fit into the DAE mould.

4. Pseudonyms and Aliases

DDK was responsible for the first mention of Bourbaki in the mathematics literature in his publication⁴ in the *Proceedings of the Academy of Sciences, UP*, in 1931, although the obscurity of the journal has resulted in the article receiving less attention than it deserved, even from a purely historical point of view. André Weil had suggested a prank, that he ascribe a theorem to a nonexistent Russian mathematician, in order to put down an older colleague in Aligarh who was giving the young Kosambi a difficult time. There is not much more

than a paragraph in Weil's autobiography⁸⁶ on this episode, so the circumstances surrounding the event are difficult to reconstruct. Nevertheless, this *parodic note passed off as a serious contribution to a provincial journal* is not entirely facetious.

It was not until December 1934 that the Bourbaki idea acquired more momentum^{109, 110}, when Weil along with Henri Cartan, Claude Chevalley, Jean Delsarte, Jean Dieudonné, and René de Possel, decided "... to define for 25 years the syllabus for the certificate in differential and integral calculus by writing, collectively, a treatise on analysis. Of course, this treatise will be as modern as possible". The book¹¹¹ would eventually appear in 1938, authored by the group that now called themselves Nicolas Bourbaki¹¹²; they then went on to write many more (and extremely influential) volumes. An Indian connection remained: when Boas mentioned (in the Britannica Book of the Year) that Bourbaki was a collective pseudonym, he got an indignant letter of protest from Bourbaki writing from his ashram in the Himalayas¹¹³. It should also be noted that Kosambi cites D. Bourbaki⁴ who is allegedly of Russian origin, while the first name eventually adopted by the Bourbaki collective¹¹⁴ is Nicolas, who is of Greek descent.

Aliases were used by DDK on several occasions although he did not use them extensively enough to warrant a distinction between his "aesthetic" or pseudonymous writing and that published under his own name. S. Ducray was merely the last nom de plume in a series, although by far the most elaborate. His first article in the magazine of Fergusson College was signed off as "Ahriman"¹¹⁵. Subsequently he wrote an expository article on the Raman effect as "Indian Scientist"¹¹⁶, and a note as "Vidyarthi"¹¹⁷: this was almost surely his nod to William Sealy Gosset, the chemist and statistician who, as "Student" invented the t-test in statistics.

It is difficult to discern what led him to use the pseudonym S. Ducray. The alleged etymology is that Bonzo, the Kosambi family dog in the 1960s, was quite plump and DDK affectionately called him *Dukker*, namely "pig" in Marathi. This evolved into Ducray, a name that sounds vaguely French, with the forename being the Sanskrit for

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dog, namely *Svana*. The choice of such a name remains enigmatic, and while it may have been prompted initially by his anger with the establishment—to date Kosambi is among the very few persons to have had their appointment terminated by the DAE—there is enough to suggest that there may be more to the use of this alias than pique.

DDK published four articles as S. Ducray, two in the *Journal of the University of Bombay*^{65, 66} and two in the *Proceedings of the Indian Academy of Sciences*^{68, 69}. The latter two were in fact communicated by C.V. Raman. While this may have been a formal device employed by the journal, it is highly unlikely that Raman knew of the masquerade. Had Raman known, it is also highly unlikely that he would have permitted such subterfuge in a journal of his Academy. These two papers were serious enough as works of mathematics, as were the other two Ducray papers that were submitted to the *Journal of the University of Bombay*. Indeed, two of these four papers were reviewed in *MR*. All the four articles show a strong connection to DDK, acknowledging him in one and quoting a private communication from Paul Erdös in another, in addition, of course, to citing his related papers written as D.D. Kosambi.

These papers continued the prime obsession that DDK showed in his last years. Regrettably, the manuscript of his book⁷¹ that was mailed to the publishers a short time before his death has never been retrieved. If nothing else, it would have provided some clues as to how he hoped to use the probability theory in this arena.

Although reviewed in *MR*, the papers had serious shortcomings. J. Kubilius who himself worked in the area of probabilistic number theory says of "Probability and prime numbers"⁶⁸ that "The reviewer could not follow the proof of the cardinal Lemma 3". The paper "Normal Sequences"⁶⁶ was comprehensively reviewed by B. Volkmann who pointed out a number of inaccuracies and misprints.

One of DDK's earlier papers had been reviewed in *MR* by E.S. Pondiczery: this was the editor Ralph Boas Jr.'s pseudonym, a fanciful "Slavic" spelling of Pondicherry. The name, which Boas used even when writing serious mathematics, was apparently concocted for

its initials, ESP, and was to have been used for writing an article debunking extra-sensory perception. Boas had a well-developed sense of the ludic and was one of the authors of the brilliant article "A Contribution to the Mathematical Theory of Big Game Hunting" that was published in the American Mathematical Monthly under the (collective) pseudonym H.W.O. Pétard¹¹⁸. Both Boas and Kosambi were publicly dismissive of extra-sensory perception, and in 1958 DDK in collaboration with U.V.R. Rao authored an article analysing the statistical defects underlying parapsychological experiments⁶⁰. This paper was subsequently commented upon by A.W. Joseph¹¹⁹ who pointed out an error in analysis as well as in the conclusions, ending with "The above comments do not detract from the valuable experiments in card-shuffling made by the authors, but it is suggested that there is little weight left in their criticism of the ESP investigations." Perhaps it was these connections that inspired Kosambi when he was to later adopt the Ducray alias.

5. Concluding Remarks

History may not have been particularly kind to Kosambi the mathematician, but in his lifetime DDK was appreciated for his scholarship and intelligence¹²⁰ early in his career, and by his peers. The manner in which Kosambi was viewed by his contemporaries—many of who were more distinguished than him and had a more significant impact in mathematics—is revealing. From 1930 to 1958 or so, DDK enjoyed the respect and admiration of a large professional circle. As has been noted earlier⁷⁵, his contributions in areas such as ancient Indian history, Sanskrit epigraphy, Indology, as well as his writings of a political and pacific nature grew both in volume and in substance in the 1940s and 1950s, overshadowing his mathematics, although the constancy of his work in the area remained. His wide scholarship and his ability to integrate different strands of thought gave him a large and dispersed audience, although his temperament and his politics were also well known and not as widely appreciated.

One important recognition that was accorded to him, in part due to his being at the TIFR and the association with Bhabha, but also for

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his work and his mathematical antecedents¹²¹, was his appointment as a member, in 1950, of the Interim Executive Committee of the International Mathematical Union, to serve along with Harald Bohr, Lars Ahlfors, Karol Borsuk, Maurice Fréchet, William Hodge, A.N. Kolmogorov and Marston Morse. One of the tasks of this rather distinguished group was to choose Fields medalists, and DDK served on this committee for two years.

It is thus noteworthy that in a period that spans three decades, Kosambi was mathematically productive, prolific, original, and was taken seriously by the scientific establishment in the country, as his elections to the Fellowships of the Indian Academy of Sciences and the Indian National Science Academy and the Presidency of the Mathematics section of the 34th Indian Science Congress in 1947, among other distinctions, testify. His papers appeared in leading journals of the world, and were communicated by or reviewed by some of the leading mathematicians of the time. And that this happened while his reputation in a diametrically different field was also burgeoning can only be seen as evidence of a complex but nevertheless Promethean intellect.

Acknowledgment

I have greatly benefited from conversations and/or correspondence with Michael Berry, S.G. Dani, Meera Kosambi, Mrs. Marston Morse, Rajaram Nityananda and Andrew Odlyzko. The TIFR archivist has been very helpful in providing copies of the correspondence between Kosambi and Bhabha, and Kapilanjan Krishan, Rahim Rajan and Mudit Trivedi have helped me obtain copies of articles by DDK that proved to be the most difficult to locate. The hospitality of the University of Tokyo where the main effort of putting together the collected mathematical works of DDK was completed (in January 2010) is gratefully acknowledged.

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⁷⁹ References Nos. 3 to 71 are a complete set of the papers of DDK that are of a mathematical nature. The list has been compiled in part from incomplete sources in the biography by Chintamani Deshmukh as well as web listings. In addition to the papers listed above, several of his essays relate to scientific issues, but these are not included here.

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⁸⁸ Sarvadaman Chowla (1907–1995) moved to the US in 1947 after a career at Delhi, Banaras, Waltair and Lahore in undivided India. A student of J.E. Littlewood, Chowla was a number theorist.

⁸⁹ In 1940, Weil was in military prison in Bonne-Nouvelle for refusing to take part in the war as a conscientious objector (since his true dharma was the pursuit of mathematics and not war, he said) when he proved an analogue of the Riemann hypothesis (for the zeta function of curves over finite fields). He did discuss the Riemann hypothesis with

T. Vijayaraghavan, who is supposed to have said that if he could have six months—undisturbed and undistracted—in a prison, he could have a crack at solving the RH. See Ref. No. 86, and M. Raynaud, 'André Weil and the Foundations of Algebraic Geometry', *Notices of the AMS* **46**, 864 (1999).

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⁹¹ The Kosambi–Bhabha correspondence has been made available through the kind courtesy of the TIFR. There are a large number of letters that are presently being catalogued and edited. Some have been reproduced in Ref. No. 74.

⁹² The most recent effort was in 2010, when Louise J. (Mrs Marston) Morse was nearly 100 years old. She was kind enough to have the Morse archives searched, but was not able to locate this manuscript or any reference to it.

⁹³ Of the 110 or so Founding Fellows, about two thirds were from the south of India or worked there and Raman might have had greater familiarity with their work or their reputation.

⁹⁴ The University of Madras announced the Ramanujan Memorial Prize for "the best thesis based on original contributions submitted by an Indian (or one domiciled in India) on some definite branch of mathematics, applied or pure" in 1933. The prize was awarded in 1934, as reported in *Nature* **135**, 28 (1935).

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⁹⁶ K.A.N., "Metrology of Punch-Marked Coins", *Current Science* 7, 345–46 (1941). This might have been the historian of south India, K.A. Nilakantha Sastry (R. Thapar, private communication).

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D. Rockmore, *Stalking the Riemann Hypothesis* (Vintage, New York, 2005).

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¹⁰⁶ In addition to Chandrashekaran and Ramanathan, the other two signatories were C.S. Seshadri and M.S. Narasimhan. Fermat's last theorem, that the equation $x^n + y^n = z^n$ has no solutions with integer x,y,z if n is larger than 2 was proved conclusively only in 1995 by Andrew Wiles (see Simon Singh, *Fermat's Enigma: The Epic Quest to Solve the World's Greatest Mathematical Problem* (Anchor Books, 1998)). Kosambi could not have had the proof he claimed in his letters to eminent mathematicians such as Carl Siegel (who was at the Institute for Advanced Study, Princeton) and others.

¹⁰⁷ D.D. Kosambi, "Lectures on Statistics", unpublished. This essay concludes, "But if I go any further into his achievements, I shall be preaching Bolshevism in the sacral portals of Bombay House and so must stop here."

¹⁰⁸ This is one of three essays on solar energy that were first reprinted in *Science, Society and Peace* (The Academy of Political and Social Studies, Pune, 1986), as well as now in Ref. No. 74.

¹⁰⁹ L. Beaulieu, "Bourbaki's Art of Memory", Osiris 14, 291 (1999).

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¹¹¹ N. Bourbaki, Éléments de Mathématique, Book 1: Théorie des

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¹¹² The history of the Bourbaki collective has been written about extensively by Maurice Mashaal, Bourbaki: *A Secret Society of Mathematicians* (American Mathematical Society, Providence, 2006) as well as others, including Liliane Beaulieu (Ref. No. 114). While the eventual name chosen by the group was Nicolas, in the original Kosambi paper, it is D. Bourbaki. Whether the initial was for Damodar, or whether this refers to another of the Bourbaki scions remains a mystery.

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¹¹⁸ H.W.O. Pétard, "A contribution to the mathematical theory of big game hunting", *American Mathematical Monthly* **45** 446 (1938).

¹¹⁹ A.W. Joseph, "A note on the paper by D.D. Kosambi and U.V. Ramamohan Rao on "The efficiency of randomization by card-shuffling", *Journal of the Royal Society of Statistics* **122**, 373–74 (1959).

¹²⁰ In "Artless innocents and ivory-tower sophisticates: Some personalities on the Indian mathematical scene", *Current Science* **85**, 526–37 (2003), M.S. Raghunathan recounts a conversation with André Weil in 1966 or 1967, when he (Weil) says of DDK, "... *Let me tell you this: he was one of the finest intellects to come out of your country.*" In his autobiography (Ref. No. 86) André Weil writes: *I appointed Kosambi for the following year. He was a young man with an original turn of mind, fresh from Harvard where he had begun to take an interest in differential geometry. I had met him in Benares (now Varanasi) where he had found a temporary position.*

¹²¹ G.D. Birkhoff, "Mathematics at Harvard in the 1940s", *Proceedings* of the American Philosophical Society **137**, 268–72 (1993).